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# Visualization of the viscous flow induced by a round jet

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The flow induced by a laminar round glycerol jet emerging from a plane wall is investigated. The flow field (jet region as well as outer flow field) is visualized by means of very small bubbles suspended in the fluid. For slender jets the observed flow pattern disagrees with both potential-flow theory and Squire's 'exact' solution, but it is in excellent agreement with results of an asymptotic analysis by Schneider (1981). At a downstream distance that depends exponentially on the square of the Reynolds number, the jet develops into a toroidal eddy as predicted by Schneider (1985). In an apparatus of reasonable size the eddy can be observed only in a very limited Reynolds-number regime.

## 1. Introduction

Viscosity effects are fundamental in the flow induced by a round jet. This was already indicated by Squire (1952) and Potsch (1981). They gave exact solutions of the Navier–Stokes equations for the problem of a round laminar jet emerging from an orifice in a plane or conical wall, but these solutions cannot satisfy the non-slip condition at the wall. Furthermore, Squire as well as Potsch showed that in the presence of walls the induced flow is not an inviscid potential flow even in the limiting case of very high orifice Reynolds number $Re_0 = M_0^{\frac{1}{2}}/\nu \rightarrow \infty$ (kinematic momentum flux at the orifice $2\pi M_0$ and kinematic viscosity $\nu$). For this case Schneider (1981) found a non-trivial similarity solution satisfying the non-slip condition at the wall, and he observed that the local Reynolds number in the induced flow is of order 1. The derived streamline pattern is substantially different from those previously published (Squire 1952; Kraemer 1971; Schlichting 1979; Potsch 1981).

In a further step of the analysis, the reactive effects of the outer flow on the jet can be investigated. Near the edge of the jet, the radial velocity component of the induced flow is negative. This gives rise to a convective momentum flux into the jet, and further contributions to the momentum balance come from pressure and viscous forces (Schneider 1985). As a consequence, the axial momentum flux in the jet is not constant as it is commonly assumed following Schlichting (1933). Rather, the momentum flux in the jet changes considerably, if slowly. Application of second-order boundary-layer theory (Mitsotakis, Schneider & Zauner 1984) leads to a breakdown of the expansion as the distance from the orifice tends to infinity. An error analysis indicates that the range of applicability of the classical first-order solution is severely limited. Schneider (1985) investigated the same problem by use of a multiple-scaling approach that accounts for the slowly varying momentum flux at the first order. It follows from the analysis that the momentum flux decreases logarithmically with increasing distance from the orifice. At a finite distance depending exponentially on $Re_0^2$ the momentum flux vanishes, the jet diameter becomes very large and a recirculatory flow is predicted, i.e. the jet is terminated. Schneider (1985) also indicated that in the recirculatory flow region the analysis is not strictly valid any more since the slender-jet assumption is violated.

Most experimental studies on laminar axisymmetric jets deal with buoyancy effects, the near-jet region, stability problems, and the transition from laminar to turbulent flow (Viilu 1962; Reynolds 1962; McNaughton & Sinclair 1966; Mollendorf & Gebhart 1973; Rankin *et al.* 1983). However, Reynolds as well as McNaughton & Sinclair, who investigated water jets in cylindrical vessels, have observed a breakdown of the jet at relatively low Reynolds numbers. Difficulties in maintaining a long steady jet in the range $4 < Re_0 < 12$ (Reynolds 1962) might have been caused not only by stability problems but also by the decay of momentum in the jet. McNaughton & Sinclair (1966) refer to a jet mushrooming in the vessel at Reynolds numbers $Re_0 < 120$ ('laminar dissipated jet'). However, density differences between the tracer solution and the bulk of the fluid may have influenced the flow.

The present paper reports experimental observations of the flow field induced by a laminar axisymmetric jet emerging from a plane wall. The experiments are carried out in two different Reynolds-number regimes. Comparison with analytical predictions (Schneider 1981, 1985) is made.

## 2. Apparatus

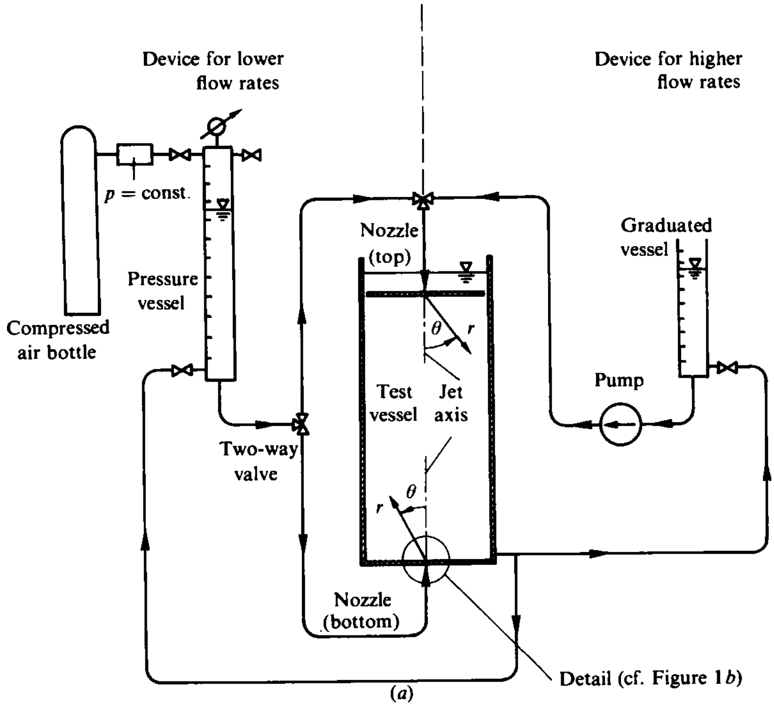
A liquid jet was injected into a vessel filled with the same liquid (figure 1*a*). The liquid was a mixture of glycerol and (about 13%) water. The vessel was made of transparent acrylic plastic and had a height of 110 cm and a cross-section of $30 \times 40\ \mathrm{cm}^2$. Two different nozzles with diameters $d_0 = 0.5$ mm and $d_0 = 1.1$ mm respectively and a length-to-diameter ratio of 10 were used. The orifices had sharp edges and were mounted flush in the wall (figure 1*b*). The parabolic velocity profile in the nozzle is developed within a length of about $5d_0$ (according to Schlichting 1979) for the largest Reynolds numbers to be applied in these experiments. The kinematic momentum flux $M_0$ at the orifice, i.e. the momentum flux referred to unit density and unit azimuthal angle, is defined by

$$M_0 = \int_0^{\frac{1}{2}d_0} u^2 y\,\mathrm{d}y, \tag{1}$$

where $u$ is the axial velocity component and $y$ is the distance from the axis of symmetry. For the parabolic velocity profile, (1) yields $M_0 = \frac{1}{6}u_0^2 d_0^2$, where $u_0$ is the volumetric mean velocity, which was determined by flow-rate measurement. Note that the orifice Reynolds number $Re_0 = M_0^{\frac{1}{2}}/\nu = u_0 d_0/\sqrt{6}\,\nu$ differs from the nozzle Reynolds number $Re_d = u_0 d_0/\nu$ by a factor $1/\sqrt{6}$.

The jets were generated by two different devices (figure 1*a*). To obtain relatively high flow rates (higher-Reynolds-number regime), the fluid was injected by a pump. For relatively low flow rates (lower-Reynolds-number regime), however, the jet fluid was supplied from a small vessel with a constant-pressure device, since under these conditions the pump did not operate in a steady state and the temperature increase due to energy dissipation was unacceptable. To estimate the influence of unavoidable, weak buoyancy effects due to dissipation in the nozzle and due to slight temperature differences between the vessel and its surroundings, experiments were performed with jets issued, alternately, from the centre of the top and from the centre of the bottom of the vessel.

To visualize the flow, small bubbles were generated in the vessel. In the higher-



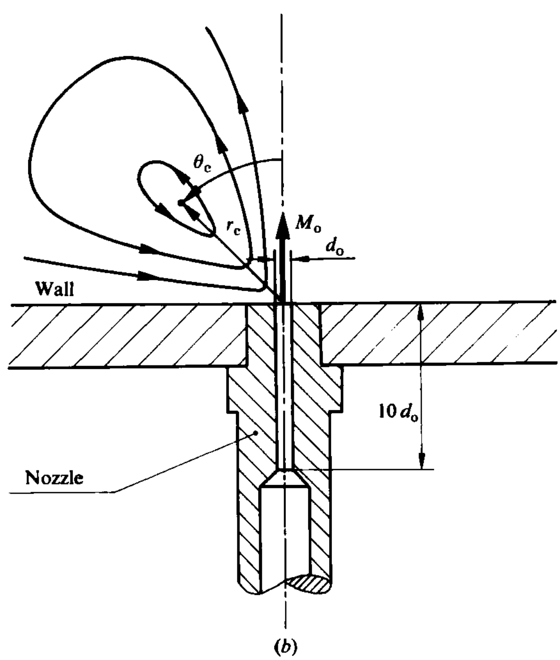




FIGURE 1. Apparatus. (*a*) Schematic set-up and coordinate system in the test section. The device on the right-hand side is used to investigate jets in the higher-Reynolds-number regime, while the left-hand side shows the device used in the lower-Reynolds-number regime. (*b*) Details of the nozzle geometry and schematic plot of the recirculatory flow pattern.

Reynolds-number regime the bubbles arose from a tiny leakage on the suction side of the pump. To observe jet flows in the lower-Reynolds-number regime, the bubbles were generated by impingement of a liquid jet issued from a nozzle positioned slightly above the free surface of the liquid in the vessel. The 'suspension' (with very low bubble concentration) was illuminated by a halogen lamp in a plane containing the jet axis. As the settling time of the bubbles was of the order of several hours, the bubbles follow the convective motion of the liquid with good accuracy. The bubble trajectories (assumed to be identical with the streamlines of the liquid flow) were photographed in the direction normal to the light plane (typical exposure time 1 minute).

## 3. Experimental programme and analytical background

The entrainment rate and, as a consequence, also the velocity of the induced flow are proportional to the kinematic viscosity $\nu$. To make the motions easily observable in an apparatus of reasonable size, the velocities must not be too low. The relatively high viscosity of the glycerol–water mixture (about 100 times larger than that of water) turned out to be advantageous. Instabilities and disturbances near the orifice, as were observed by Viilu (1962) and Reynolds (1962) at low Reynolds numbers, did not occur in our experiments. This is in line with the stability analysis by Mollendorf & Gebhart (1973), which shows that the spatial amplification rate of the disturbance amplitude increases with decreasing $\nu$.

In the first part of the study, the flow induced by the jet was investigated at Reynolds numbers and distances from the orifice chosen such that the effect of the decay of momentum flux in the jet was negligible. Under these conditions the jet extended over the whole length of the vessel. This, of course, resulted in a reverse flow outside the jet region. Therefore only a field of 5 cm diameter (i.e. about one eighth of the vessel width) was subject to observation.

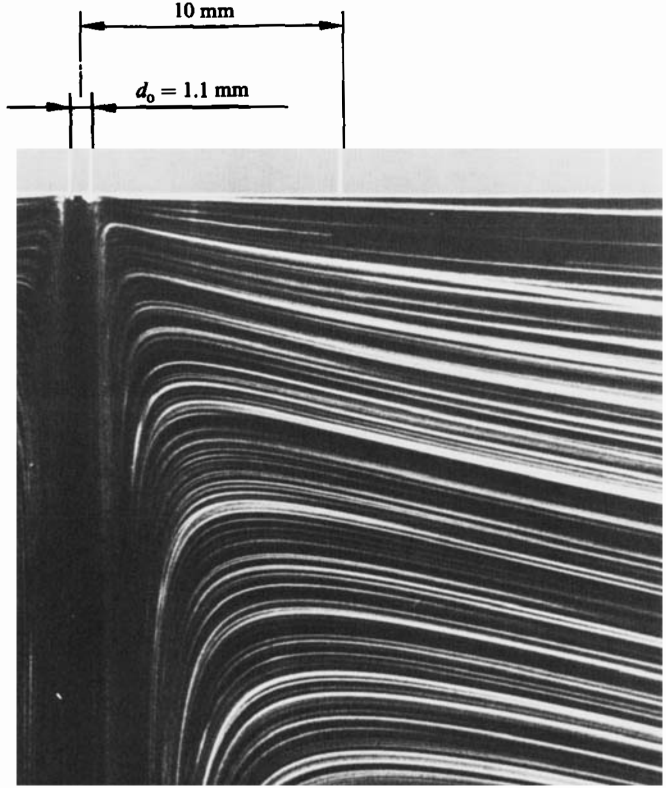
The second part of the study was concerned with the reactive effects of the induced flow on the jet. As already mentioned, an analysis due to Schneider (1985) predicts a logarithmic decay of momentum flux in the jet and a recirculatory flow region, i.e. a viscous toroidal eddy. According to the analysis in polar coordinates, the location of the centre of the eddy is given by the radial distance $r_c$ from the orifice and the angular coordinate $\theta_c$, where

$$\frac{r_c}{d_0} = A \exp\left(\frac{Re_0^2}{8C^2}\right), \tag{2}$$

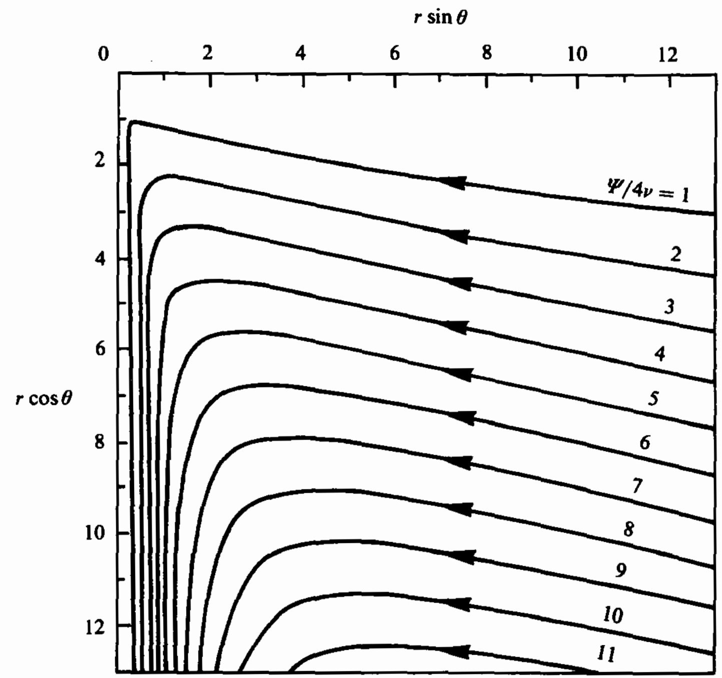
while $\theta_c$ is independent of $Re_0$. Both $\theta_c$ and $C^2$ (the latter characterizing the strength of the momentum decay) depend on the wall geometry. For a plane wall, $C^2 = 1.91$ and $\theta_c = 45°$ are the values predicted by the asymptotic analysis (Schneider 1985). The dimensionsless coefficient $A$ depends on the flow characteristics near the nozzle such as the velocity profile and the geometry of the orifice. Since the analysis ceases to be valid in this region, $A$ (which is supposed to be of the order of 1, i.e. independent of $Re_0$) remains undetermined. From (2) it can be seen that for the given size of the vessel only Reynolds numbers of rather limited magnitude can be investigated.

## 4. Results

Figure 2($a$) shows the flow induced by a laminar round jet emerging from a plane wall with $Re_0 = 32.6$. It can be seen very clearly that, owing to viscosity effects, the flow is displaced from the wall. The observations are compared with the results of




(a)




(b)

FIGURE 2. Streamlines of a round laminar jet flow emerging from an orifice in a plane wall perpendicular to the jet axis. (*a*) Flow visualization; $Re_o = 32.6$ ($d_o = 1.1$ mm, vessel size $0.3 \times 0.4 \times 1.1$ m, exposure time 64 s). (*b*) Similarity solution with constant momentum flux; $Re_o = 30$ (composite first approximation due to Schneider (1981), arbitrary unit of length).

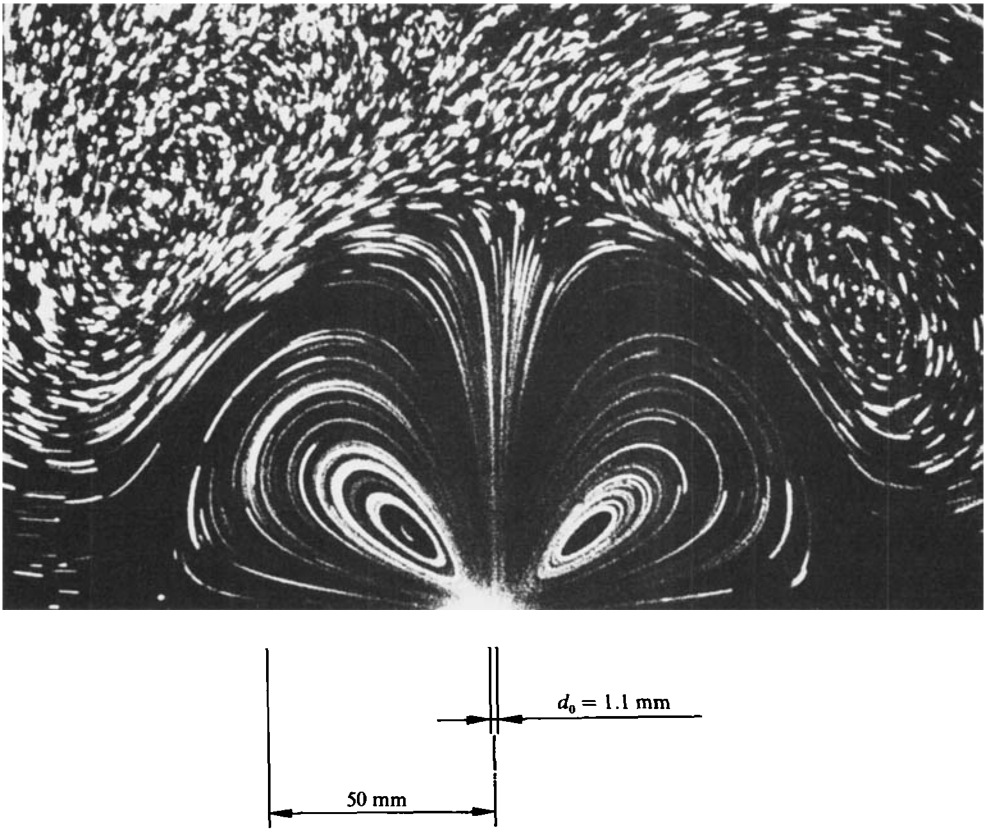



FIGURE 3. Recirculatory flow induced by a round jet; $Re_0 = 7$ ($d_0 = 1.1$ mm, issued from the bottom, exposure time 64 s).

the similarity solution due to Schneider (1981). In the induced-flow region the predicted streamlines (figure 2*b*) agree with the observed ones (figure 2*a*) without any visible discrepancies. This is in contrast with potential-flow theory, which predicts streamlines that are parallel to the wall because the jet acts as a line sink with constant entrainment rate. The streamlines resulting from the 'exact' solutions due to Squire (1952) and Potsch (1981) would pass even closer to the wall than according to potential-flow theory.

Observations of jets in the lower-Reynolds-number regime indicate a completely different flow pattern. Figure 3 presents the recirculatory flow at $Re_0 = 7$, which is in good qualitative agreement with theoretical predictions (Schneider 1985). The recirculatory streamline pattern was observed even when artificially introduced buoyancy effects caused the jet to behave like a plume downstream of the eddy. Thus there seems to be no doubt regarding the existence of the viscous eddy. However, the present experiments cannot give reliable information regarding the structure of the flow field at distances from the orifice that are large compared with $r_c$. This is due to the facts that the jet was injected into a confined region (rather than into a semi-infinite space) and that weak buoyancy effects could not be completely prevented in our apparatus.

The location of the centre of the viscous eddy (for the coordinate system cf. figure 1*b*) determined from flow-visualization photographs is shown in figures 4(*a*, *b*). With

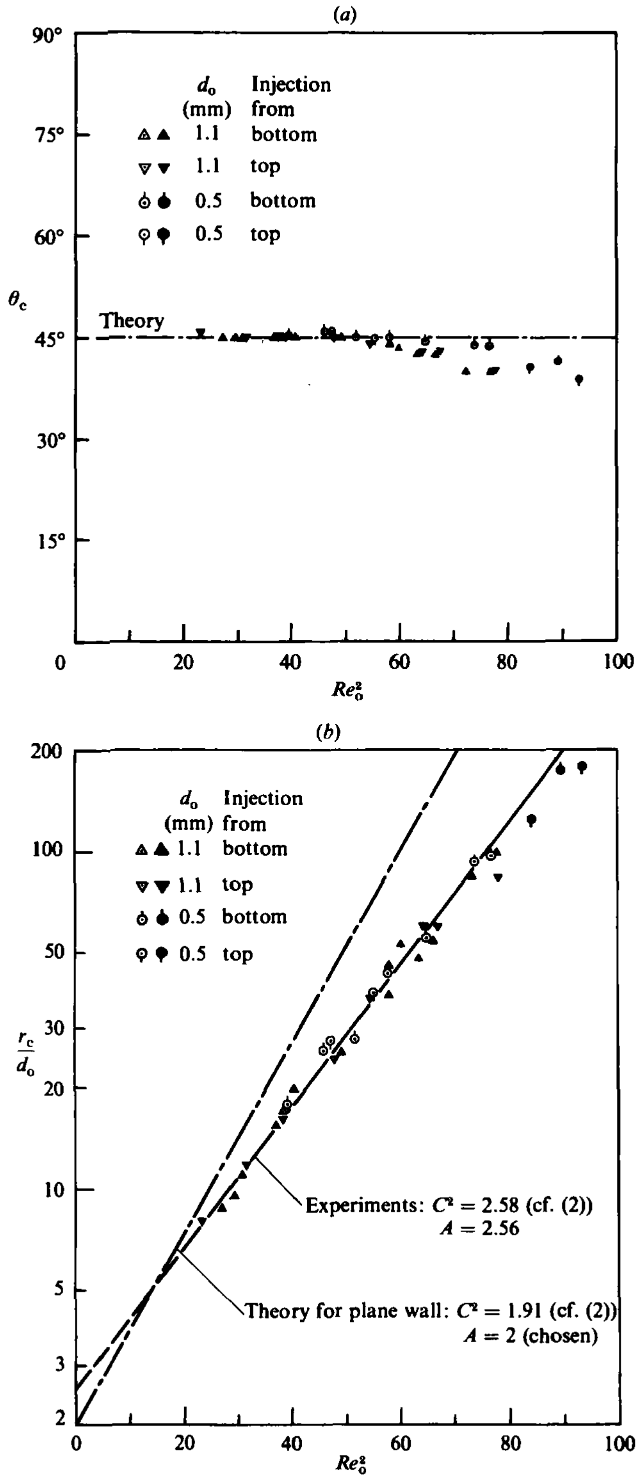



FIGURE 4. Location of the centre of the viscous eddy versus orifice Reynolds number $Re_0$. △, ▽, ⊙, ⊙, data with $r_c < 5$ cm (sidewall effect negligible); ▲, ▼, ●, ●, data with $r_c > 5$ cm; — - —, asymptotic analysis (Schneider 1985). (*a*) Radial distance from the centre of the orifice. (*b*) Angular coordinate measured from the jet axis.

respect to the angular coordinate $\theta_c$ (figure 4*a*) the results are independent of the nozzle diameter for Reynolds numbers up to $Re_0^2 \approx 60$ (i.e. $Re_0 \approx 7.7$); the deviations are less than 1°. For larger Reynolds numbers the differences between the results for different nozzle diameters become noticeable, presumably owing to the disturbing effect of the sidewalls. The radial distance $r_c$ (figure 4*b*) corresponding to $Re_0 = 7.7$ is about 5 cm (i.e. approximately a quarter of the vessel half-width) in the case of the larger nozzle diameter ($d_0 = 1.1$ mm). This gives an upper limit for the ratio of radial distance $r_c$ and vessel half-width (approx. 0.25) up to which the disturbing effect of the sidewalls can be neglected. The data that are below the limit are represented by open symbols in the figures, whereas the filled symbols represent the results exceeding the limit.

The comparison of the measured angular coordinate $\theta_c$ with the theoretical prediction (figure 4*a*) shows good agreement for the data below the limit.

Regarding the results for the dimensionless radial distance $r_c/d_0$ from the orifice (figure 4*b*), the data (below the limit) obtained with both nozzle diameters as well as both nozzle positions show little scatter. This confirms that the nozzles were geometrically similar, and it ensures that the above-mentioned uncertainties due to buoyancy effects did not have a significant influence on the flow in this region. Since the results (below the limit) can be approximated by a straight line in the $\ln(r_c/d_0)$ versus $Re_0^2$ plot, the experiments indicate an exponential growth of $r_c$ with $Re_0^2$ as predicted by the analysis (Schneider 1985), cf. (2). Furthermore, the results seem to confirm the logarithmic decay of momentum flux in the round jet. Application of least-square linear regression yields the values 2.58 and 2.56 respectively for the parameters $C^2$ and $A$ in (2). The relatively large difference between the values of the parameter $C^2$ determined from experiment ($C^2 = 2.58$) and from analysis ($C^2 = 1.91$) respectively could be due to the following.

(*a*) There are uncertainties in the determination of the orifice Reynolds number because of errors in nozzle diameter (approx. $\pm 5\,\%$), flow-rate measurement (approx. $\pm 2\,\%$) and viscosity measurement (approx. $\pm 3\,\%$).

(*b*) The asymptotic analysis (Schneider 1985) is not strictly valid in this region. Furthermore, the recirculatory flow, predicted by the analysis, is not taken into account in the momentum balance from which the parameter $C^2$ is determined.

## 5. Conclusions

Laminar axisymmetric jets issued from a large plane wall were observed by flow visualization. The main results are as follows.

(*a*) The experiments illustrate the significance of viscosity effects in the outer flow. The results confirm the asymptotic theory of Schneider (1981) for sufficiently large Reynolds numbers (i.e. the location of the centre of the eddy is far outside the observed flow field).

(*b*) At lower Reynolds numbers (with the upper limit given by the size of the apparatus) a viscous toroidal eddy was observed. Regarding the distance $r_c$ of the centre of the eddy from the orifice, the experimental data provide support for an exponential relationship between $r_c$ and $Re_0^2$ as predicted by Schneider (1985).

Some details of the flow field at distances that are large compared with $r_c$ require further investigations.

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